

Imperfect and Unmatched CSIT is Still Useful for the Frequency Correlated MISO Broadcast Channel

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Abstract—Since Maddah-Ali and Tse showed that the completely stale transmitter-side channel state information (CSIT) still benefits the Degrees of Freedom (DoF) of the Multiple-Input-Multiple-Output (MISO) Broadcast Channel (BC), there has been much interest in the academic literature to investigate the impact of imperfect CSIT on *DoF* region of time correlated broadcast channel. Even though the research focus has been on time correlated channels so far, a similar but different problem concerns the frequency correlated channels. Indeed, the imperfect CSIT also impacts the *DoF* region of frequency correlated channels, as exemplified by current multi-carrier wireless systems.

This contribution, for the first time in the literature, investigates a general frequency correlated setting where a two-antenna transmitter has imperfect knowledge of CSI of two single-antenna users on two adjacent subbands. A new scheme is derived as an integration of Zero-Forcing Beamforming (ZFBF) and the scheme proposed by Maddah-Ali and Tse. The achievable *DoF* region resulted by this scheme is expressed as a function of the qualities of CSIT.¹

I. INTRODUCTION

In downlink multi-user multiple-input-multiple-output (MIMO) communications, the latency and inaccurate CSIT degrade the *DoF* when conventional precoding techniques such as ZFBF are employed. Strategies to exploit imperfect feedback to enhance *DoF* region has therefore attracted a lot of attention. The completely stale CSIT was first studied by Maddah-Ali and Tse. In their contribution [1], an optimal per-user *DoF* of $\frac{2}{3}$ in a two-user setup was achieved by a simple transmission scheme (denoted as MAT scheme in the sequel). That result was extended later in [2] and [3], by accounting for imperfect current CSIT. The optimal *DoF* was derived and expressed as a function of the quality of the current CSIT. The achievability was shown using a scheme that bridges the *DoF* found in [1] and [2][3].

In [4] and [5], unequal quality of current CSIT per user was investigated. The optimal bound found is a superset of the results in [2] and [3], revealing that an asymmetric *DoF* is achieved by each user. Moreover, [6] has studied the imperfect delayed CSIT, suggesting that it can be used as good as perfect delayed CSIT.

To date, all the works only focus on the time correlated channel. However, the *DoF* region of frequency correlated

channels is also impacted by the imperfect CSIT. In current multi-carrier communication systems, the CSIT is measured and reported by users on a per-subband basis. In practice, each user only reports its CSI on a group of predefined subbands, which might provide few information about the channel of the subbands outside the group because of the weak correlation between different subbands. In this paper, we investigate, for the first time in the literature, the *DoF* of a general two-subband based frequency correlated broadcast channel with arbitrary imperfect CSIT (see Section II). Our contributions are summarized as follows:

- 1) Derive an achievable *DoF* of a two-user and two-subband based scenario as a function of the quality of the CSIT,
- 2) Design a novel transmission strategy, motivated by MAT and ZFBF, that achieves the *DoF* region.

The rest of this paper is organized as follows. The system model is introduced in Section II and the achievable *DoF* region is given in Section III. The *DoF* achieved via reusing MAT and ZFBF is identified in Section IV and a novel transmission scheme is introduced. Section V concludes the paper.

The following notations are used throughout the paper. Bold lower letters stand for vectors whereas a symbol not in bold font represents a scalar. $(\cdot)^T$ and $(\cdot)^H$ represent the transpose and conjugate transpose of a matrix or vector respectively. \mathbf{h}^\perp denotes the orthogonal space of the channel vector \mathbf{h} . $\mathcal{E}[\cdot]$ refers to the expectation of a random variable, vector or matrix. $\|\cdot\|$ is the norm of a vector. $|\cdot|$ represents the magnitude of a scalar. $f(P) \sim P^B$ corresponds to $\lim_{P \rightarrow \infty} \frac{\log f(P)}{\log P} = B$, where P is supposed to be the SNR throughout the paper and logarithms are in base 2. P_a denotes the power of a while $R_a^{(1)}$ and $R_a^{(2)}$ represent the rate of a achieved at receiver 1 and 2 respectively.

II. SYSTEM MODEL

We consider a two-user broadcast channel with two transmit antennas and one antenna per user. The related parameters are defined as follows. \mathbf{h}_i and \mathbf{g}_i are the channel states in subband i of user 1 and user 2 respectively. Denoting the transmit signal vector in subband i as \mathbf{s}_i , subject to a per-subband based power constraint $\mathcal{E}[\|\mathbf{s}_i\|^2] \sim P$, the observations at receiver 1 and 2,

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	User 1	User 2
Subband 1	β	α
Subband 2	α	β
	\vdots	\vdots

Fig. 1: The two-subband based scenario with imperfect CSIT.

y_i and z_i respectively, can be written as

$$y_i = \mathbf{h}_i^H \mathbf{s}_i + \epsilon_{i,y}, \quad (1)$$

$$z_i = \mathbf{g}_i^H \mathbf{s}_i + \epsilon_{i,z}, \quad (2)$$

where $\epsilon_{i,y}$ and $\epsilon_{i,z}$ are unit power AWGN noise. Signal vector \mathbf{s}_i is a function of the symbol vectors for user 1 and user 2, denoted as \mathbf{u}_i and \mathbf{v}_i respectively. \mathbf{u}_i is a two-element symbol vector containing $u_{i,1}$ and $u_{i,2}$. Similarly, \mathbf{v}_i is defined.

The channels are characterized as follows. \mathbf{h}_i and \mathbf{g}_i are mutually independent and identically distributed with zero mean and unit covariance matrix ($\mathcal{E}[\mathbf{h}_i^H \mathbf{g}_i] = 0$ and $\mathcal{E}[\mathbf{h}_i \mathbf{h}_i^H] = \mathbf{I}_2$). The imperfect CSIT of user 1 is denoted as $\hat{\mathbf{h}}_i$ while the imperfect CSIT of user 2 is $\hat{\mathbf{g}}_i$, each with the error vector of $\tilde{\mathbf{h}}_i = \mathbf{h}_i - \hat{\mathbf{h}}_i$ and $\tilde{\mathbf{g}}_i = \mathbf{g}_i - \hat{\mathbf{g}}_i$. The variances of the error vectors are $\mathcal{E}[\|\tilde{\mathbf{h}}_i\|^2] = \sigma_{h,i}^2$ and $\mathcal{E}[\|\tilde{\mathbf{g}}_i\|^2] = \sigma_{g,i}^2$.

The CSIT setting in this two-subband based scenario is illustrated in Figure 1. User 1 estimates its channel information in the first subband using pilots and feeds it back as $\hat{\mathbf{h}}_1$ while user 2 reports its CSI in the second subband as $\hat{\mathbf{g}}_2$. As shown, we assume the qualities of $\hat{\mathbf{h}}_1$ and $\hat{\mathbf{g}}_2$ are identical and expressed using a parameter, β , which is defined as

$$\beta \triangleq \lim_{P \rightarrow \infty} -\frac{\log \sigma_{h,1}^2}{\log P} = \lim_{P \rightarrow \infty} -\frac{\log \sigma_{g,2}^2}{\log P}. \quad (3)$$

As the CSI of two adjacent subbands are correlated, the transmitter can predict the channel information of the unreported subband. To be specific, with the knowledge of $\hat{\mathbf{h}}_1$, the channel condition of the second subband of user 1, $\hat{\mathbf{h}}_2$, is predicted. Similarly, $\hat{\mathbf{g}}_1$ is predicted based on the knowledge of $\hat{\mathbf{g}}_2$. The qualities of these two predicted channel states are characterized as α , which is defined as

$$\alpha \triangleq \lim_{P \rightarrow \infty} -\frac{\log \sigma_{h,2}^2}{\log P} = \lim_{P \rightarrow \infty} -\frac{\log \sigma_{g,1}^2}{\log P}. \quad (4)$$

Remarks: 1) β and α vary within the range of $[0,1]$, where 0 represents no CSIT whereas 1 stands for perfect CSIT; 2) The quality of the predicted CSIT, $\hat{\mathbf{h}}_2$ and $\hat{\mathbf{g}}_1$, is bounded by the quality of $\hat{\mathbf{h}}_1$ and $\hat{\mathbf{g}}_2$, namely $\alpha \leq \beta$; 3) We assume that this two-subband scenario can be repeated an infinite number of times; 4) The transmitter and both users have the knowledge of $\hat{\mathbf{h}}_{1:2N}$ and $\hat{\mathbf{g}}_{1:2N}$. Besides, each receiver has perfect knowledge of local CSI; 5) It is important to note the quantities $\mathcal{E}[\|\mathbf{h}_1^H \hat{\mathbf{h}}_1^\perp\|^2] = \mathcal{E}[\|\mathbf{g}_2^H \hat{\mathbf{g}}_2^\perp\|^2] \sim P^{-\beta}$ and $\mathcal{E}[\|\mathbf{g}_1^H \hat{\mathbf{g}}_1^\perp\|^2] = \mathcal{E}[\|\mathbf{h}_2^H \hat{\mathbf{h}}_2^\perp\|^2] \sim P^{-\alpha}$.

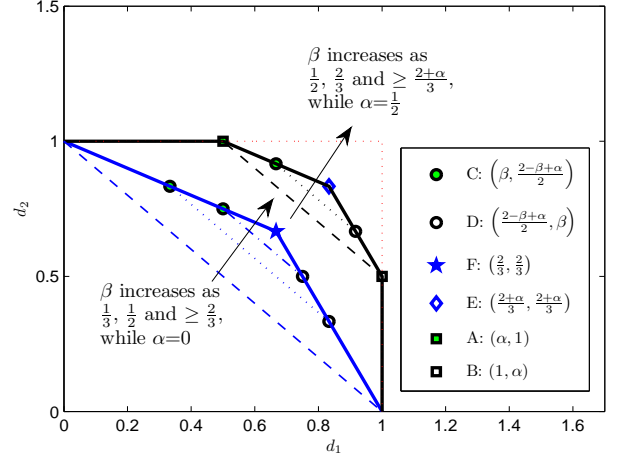


Fig. 2: Achievable DoF region in frequency correlated channel with imperfect CSIT.

Throughout the paper, we define a per-channel-use based DoF, which is expressed as

$$d_i \triangleq \lim_{P \rightarrow \infty} \frac{R_i}{S \log P}, \quad i = 1, 2, \quad (5)$$

where R_i is the rate achieved by user i over S channel uses².

III. DoF REGION WITH GENERAL CSIT PATTERN

Theorem 1. In a frequency correlated MISO BC with imperfect CSIT shown in Figure 1, an achievable DoF region is characterized as a polygon composed of the following corner points:

$$\begin{aligned} & \{(1,0); (0,1); (1,\alpha); (\alpha,1); \dots \\ & \left(\min \left(\frac{2+\alpha}{3}, \beta \right), \max \left(\frac{2+\alpha}{3}, \frac{2-\beta+\alpha}{2} \right) \right); \dots \\ & \left(\max \left(\frac{2+\alpha}{3}, \frac{2-\beta+\alpha}{2} \right), \min \left(\frac{2+\alpha}{3}, \beta \right) \right) \}. \end{aligned} \quad (6)$$

Figure 2 illustrates the region specified by (6), spanning all β and α satisfying $\beta, \alpha \in [0,1]$ and $\alpha \leq \beta$. When α is fixed, points C and D (shown by circle points, see Figure 2) move closer to each other as β increases. For $\beta = \frac{2+\alpha}{3}$, points C and D join at point E (or F, see Figure 2). If β continues increasing, the DoF region will not expand any further. This reveals that the CSIT with quality β satisfying $\beta \geq \frac{2+\alpha}{3}$ can be as good as $\beta=1$. Specifically, the DoF region achieved by MAT [1] with $\beta=1$ and $\alpha=0$ (composed of point F, (1,0) and (0,1), see Figure 2) can be actually achieved by $\beta = \frac{2}{3}$ and $\alpha=0$.

Moreover, if we fix β and increase α , all the points will move either upwards or to the right. When α reaches β , point A will join point C while points B and D overlap. The DoF region can be simply achieved by doing ZFBF plus superposition coding.

² S channel uses may physically refer to S subbands with full transmission power P or transmitting symbols using power of P^S in a single subband. In this paper, it takes the latter understanding.

In addition, the maximum sum *DoF* is achieved by the diamond point E, and star point F, for the case $\beta \geq \frac{2+\alpha}{3}$. Otherwise, it is obtained by the circle points C and D.

IV. ACHIEVABILITY

A. Motivations

In this part, we briefly revisit two existing schemes, MAT and ZFBF. Their achievable rates in frequency correlated BC will be identified and analyzed. Their sub-optimality will motivate the derivation of the novel transmission strategy.

1) *Reusing MAT scheme and Extensions*: In [1], the transmission of MAT finishes in three time slots, during which the transmit signal and received signals are

$$\begin{aligned} \mathbf{s}_1 &= \mathbf{v}_1, & \mathbf{s}_2 &= \mathbf{u}_2, & \mathbf{s}_3 &= [\eta_{1,1} + \eta_{2,2}, 0]^T, \\ y_1 &= \eta_{1,1}, & y_2 &= \mathbf{h}_2^H \mathbf{u}_2, & y_3 &= h_{3,1}^* (\eta_{1,1} + \eta_{2,2}), \\ z_1 &= \mathbf{g}_1^H \mathbf{v}_1, & z_2 &= \eta_{2,2}, & z_3 &= g_{3,1}^* (\eta_{1,1} + \eta_{2,2}), \end{aligned}$$

where $\eta_{1,1} = \mathbf{h}_1^H \mathbf{v}_1$ and $\eta_{2,2} = \mathbf{g}_2^H \mathbf{u}_2$. User 1 receives its desired symbol vector \mathbf{u}_2 in y_2 but overhears $\mathbf{h}_1^H \mathbf{v}_1$ in y_1 . The decoding is enabled once the transmission at slot 3 is completed, where the sum of the overheard interference is retransmitted. After decoding $\eta_{1,1} + \eta_{2,2}$ received in y_3 and subtracting y_1 , user 1 obtains an additional independent observation of its desired symbol vector, $\mathbf{g}_2^H \mathbf{u}_2$. Hence, user 1 can decode \mathbf{u}_2 . Similarly, user 2 can decode \mathbf{v}_1 . In this way, four symbols are successfully transmitted in three slots, resulting in the symmetric *DoF* of $\frac{2}{3}$.

However, among all the six CSI in these three time slots, only two of them are in fact employed, namely \mathbf{h}_1^H and \mathbf{g}_2^H . Equivalently, we can reuse MAT in the scenario shown in Figure 1 provided that $\beta=1$ and $\alpha=0$. The sum of the overheard interference, $\eta_{1,1} + \eta_{2,2}$, is reconstructed and retransmitted using an extra channel use (subband 3). The CSI of this extra channel use does not have to be known at the transmitter.

When $\beta < 1$, the transmit power should be adjusted because the overheard interferences generated at each user are reconstructed with non-negligible error at the transmitter. Specifically, after subtracting y_1 from $y_3/h_{3,1}^*$, $\hat{\mathbf{g}}_2^H \mathbf{u}_2$ is obtained plus a residue interference, $(\hat{\mathbf{h}}_1^H - \mathbf{h}_1^H) \mathbf{v}_1 = -\tilde{\mathbf{h}}_1^H \mathbf{v}_1$, where $\mathcal{E}[\|\tilde{\mathbf{h}}_1^H\|^2] \sim P^{-\beta}$. To make the residue interference drowned by the noise, the transmission power of \mathbf{v}_1 in subband 1 should be reduced to P^β . In this way, β channel use is employed per subband, during which, both \mathbf{v}_1 and \mathbf{u}_2 achieve the rate $2\beta \log P$ resulting in the sum *DoF* $\frac{4\beta}{3}$ over 3β channel uses.

2) *Conventional approach-ZFBF*: ZFBF is one of the conventional interference mitigation techniques that achieve MIMO transmission. The transmitter precodes two symbols u_1 and v_1 (intended to user 1 and 2 respectively) using the knowledge of CSIT of both users. The transmission signal in subband 1 is expressed as

$$\mathbf{s}_1 = \hat{\mathbf{g}}_1^\perp u_1 + \hat{\mathbf{h}}_1^\perp v_1, \quad (7)$$

where $P_{u_1} \sim P^\alpha$ and $P_{v_1} \sim P^\beta$, resulting in the received signals

$$y_1 = \mathbf{h}_1^H \hat{\mathbf{g}}_1^\perp u_1 + \mathbf{h}_1^H \hat{\mathbf{h}}_1^\perp v_1 + \epsilon_{1,y}, \quad (8)$$

$$z_1 = \mathbf{g}_1^H \hat{\mathbf{g}}_1^\perp u_1 + \mathbf{g}_1^H \hat{\mathbf{h}}_1^\perp v_1 + \epsilon_{1,z}. \quad (9)$$

As the qualities of $\hat{\mathbf{h}}_1$ and $\hat{\mathbf{g}}_1$ are β and α respectively, $\mathbf{h}_1^H \hat{\mathbf{h}}_1^\perp v_1$ and $\mathbf{g}_1^H \hat{\mathbf{g}}_1^\perp u_1$ are drowned by the noise. In this way, the rate achieved by u_1 and v_1 are $\alpha \log P$ and $\beta \log P$ respectively. The amount of channel use in subband 1 is β . Similarly, the same transmission is applicable in subband 2 by switching the power of each user's symbol. Hence, the sum *DoF* is $\frac{2\beta+2\alpha}{2\beta}$ during these 2β channel uses.

	Sum Rate (log P)	Channel Uses	$n_{1,1}, n_{1,2}$	$n_{2,1}, n_{2,2}$	Pre- coding
MAT	4β	3β	0, 2	2, 0	No
ZFBF	$2\beta+2\alpha$	2β	1, 1	1, 1	Yes
Objective	4β	$< 3\beta$	1, 2	2, 1	Yes

TABLE I: Comparison among MAT, ZFBF and the objective. $n_{i,j}$ refers to the number of private symbols sent to user i in subband j .

3) *Analysis and motivation*: The comparison between MAT and ZFBF are presented in Table I. ZFBF saves β channel uses while it incurs a rate loss of $2(\beta-\alpha) \log P$ compared to MAT.

When α is small, MAT outperforms ZFBF in sum *DoF*. In this case, ZFBF precoding works inefficiently in rejecting the interference potentially seen by user 2 in subband 1, the transmit power of u_1 in (7) is therefore significantly limited, resulting in low *DoF*. Similarly, user 2 achieves low rate in subband 2. However, MAT transmits two symbols to each user in turn. The CSIT with quality β is exploited to provide confident side information over an extra channel use. The *DoF* is therefore boosted up.

When α approaches β , ZFBF works well in rejecting the interference potentially seen by both users in each subband. The sum rate achieved by ZFBF therefore approximates as $4\beta \log P$, resulting in a higher sum *DoF* than MAT by saving the β extra channel uses. However, MAT incurs a loss because the CSIT with quality α is wasted during the 3β channel uses.

Intuitively, given a certain value of $\alpha \in [0, \beta]$, a better sum *DoF* can be obtained by a strategy that optimally balances the employment of CSIT with quality α and the usage of extra channel use. This objective strategy can be designed as the integration of ZFBF and MAT. It would outperform ZFBF by employing a small fraction of extra channel use to perform overheard interference cancellation. At the same time, as precoding is introduced, the amount of extra channel use could be reduced compared to MAT while the sum rate remains $4\beta \log P$. The amount of extra channel use would be a function of β and α , bridging ZFBF and MAT. When $\alpha = \beta$, the transmission scheme would be upgraded to ZFBF; for $\alpha = 0$, it would collapse to pure MAT. Bearing this in mind, we derive the novel transmission block in the following section.

B. Building New Transmission Blocks

Following the aforementioned motivation, the main features of this strategy are presented in the last row of Table I. It is

a combination of ZFBF and MAT in terms of precoding and the number of symbols transmitted to each user per subband.

The transmission signals in subband 1 and 2 are respectively expressed as

$$\mathbf{s}_1 = [x_{c,1}, 0]^T + [\mu_1, 0]^T + [\hat{\mathbf{h}}_1^\perp, \hat{\mathbf{h}}_1]^T \mathbf{v}_1 + \hat{\mathbf{g}}_1^\perp u_1, \quad (10)$$

$$\mathbf{s}_2 = [x_{c,2}, 0]^T + [\mu_2, 0]^T + [\hat{\mathbf{g}}_2^\perp, \hat{\mathbf{g}}_2]^T \mathbf{u}_2 + \hat{\mathbf{h}}_2^\perp v_2, \quad (11)$$

where two private symbols ($\mathbf{v}_1 = [v_{1,1}, v_{1,2}]^T$) are transmitted to user 2 and one private symbol (u_1) is sent to user 1 in subband 1. Precoding is also considered in \mathbf{s}_1 , where $v_{1,2}$ is precoded with $\hat{\mathbf{h}}_1$, $v_{1,1}$ and u_1 are projected to the orthogonal space of $\hat{\mathbf{h}}_1$ and $\hat{\mathbf{g}}_1$ respectively. The new symbols in \mathbf{s}_2 are similarly encoded and transmitted.

Besides, μ_1 and μ_2 are two pieces of the quantized overheard interference, encoded with the rates, $R_{\mu_1} = r_{\mu_1} \log P$ and $R_{\mu_2} = r_{\mu_2} \log P$ respectively. $x_{c,1}$ and $x_{c,2}$ are common messages that should be decoded by both users. The power and rate allocation are presented in Table II.

	Symbols	Power	Encoding rate ($\log P$)
Subband 1	$x_{c,1}$	$P - P^{r_{\mu_1} + \beta}$	$1 - r_{\mu_1} - \beta$
	μ_1	$P^{r_{\mu_1} + \beta} - P^\beta$	r_{μ_1}
	$v_{1,1}$	$P^\beta / 2$	β
	$v_{1,2}$	$P^\beta / 2 - P^\alpha / 2$	$\beta - \alpha$
	u_1	$P^\alpha / 2$	α
Subband 2	$x_{c,2}$	$P - P^{r_{\mu_2} + \beta}$	$1 - r_{\mu_2} - \beta$
	μ_2	$P^{r_{\mu_2} + \beta} - P^\beta$	r_{μ_2}
	$u_{2,1}$	$P^\beta / 2$	β
	$u_{2,2}$	$P^\beta / 2 - P^\alpha / 2$	$\beta - \alpha$
	v_2	$P^\alpha / 2$	α

TABLE II: The power and rate allocated to the symbols in (10) and (11).

The received signals at each receiver in each subband are expressed as

$$y_1 = \underbrace{h_{1,1}^* x_{c,1}}_P + \underbrace{h_{1,1}^* \mu_1}_{P^{r_{\mu_1} + \beta}} + \underbrace{\eta_{1,1}}_{P^\beta} + \underbrace{\mathbf{h}_1^H \hat{\mathbf{g}}_1^\perp u_1}_{P^\alpha} + \epsilon_{1,y}, \quad (12)$$

$$z_1 = \underbrace{g_{1,1}^* x_{c,1}}_P + \underbrace{g_{1,1}^* \mu_1}_{P^{r_{\mu_1} + \beta}} + \underbrace{\mathbf{g}_1^H [\hat{\mathbf{h}}_1^\perp, \hat{\mathbf{h}}_1] \mathbf{v}_1}_{P^\beta} + \underbrace{\mathbf{g}_1^H \hat{\mathbf{g}}_1^\perp u_1}_{P^0} + \epsilon_{1,z}, \quad (13)$$

$$y_2 = \underbrace{h_{2,1}^* x_{c,2}}_P + \underbrace{h_{2,1}^* \mu_2}_{P^{r_{\mu_2} + \beta}} + \underbrace{\mathbf{h}_2^H [\hat{\mathbf{g}}_2^\perp, \hat{\mathbf{g}}_2] \mathbf{u}_2}_{P^\beta} + \underbrace{\mathbf{h}_2^H \hat{\mathbf{h}}_2^\perp v_2}_{P^0} + \epsilon_{2,y}, \quad (14)$$

$$z_2 = \underbrace{g_{2,1}^* x_{c,2}}_P + \underbrace{g_{2,1}^* \mu_2}_{P^{r_{\mu_2} + \beta}} + \underbrace{\eta_{2,2}}_{P^\beta} + \underbrace{\mathbf{g}_2^H \hat{\mathbf{h}}_2^\perp v_2}_{P^\alpha} + \epsilon_{2,z}, \quad (15)$$

where $\eta_{1,1}$ and $\eta_{2,2}$ are the overheard interferences generated at user 1 in subband 1 and at user 2 in subband 2 respectively,

$$\eta_{1,1} = \underbrace{\mathbf{h}_1^H \hat{\mathbf{h}}_1^\perp v_{1,1}}_{P^0} + \underbrace{\mathbf{h}_1^H \hat{\mathbf{h}}_1 v_{1,2}}_{P^\beta}, \quad (16)$$

$$\eta_{2,2} = \underbrace{\mathbf{g}_2^H \hat{\mathbf{g}}_2^\perp u_{2,1}}_{P^0} + \underbrace{\mathbf{g}_2^H \hat{\mathbf{g}}_2 u_{2,2}}_{P^\beta}. \quad (17)$$

Note that the power stated below each term is obtained asymptotically, which is merely valid at high SNR. Since

$\mathcal{E} [|\mathbf{h}_1^H \hat{\mathbf{h}}_1^\perp|^2] \sim P^{-\beta}$, the term $\mathbf{h}_1^H \hat{\mathbf{h}}_1^\perp v_{1,1}$ in $\eta_{1,1}$ is drowned by the noise in (12). Similarly, $\mathbf{g}_2^H \hat{\mathbf{g}}_2^\perp u_{2,1}$ in $\eta_{2,2}$ vanishes.

Hence, the overheard interference $\eta_{1,1}$ and $\eta_{2,2}$ are composed of $v_{1,2}$ and $u_{2,2}$ respectively, which are then possible to be detected at receiver 1 and 2 respectively (will be discussed in Section IV-C). In this way, when reconstructing the sum of the overheard interferences, the channel component, for instance, $\mathbf{h}_1^H \hat{\mathbf{h}}_1$ in (16), can be dropped. As a consequence, in contrast to MAT where $\eta_{1,1} + \eta_{2,2}$ is rebuilt and sent, we reconstruct the sum of the symbols $v_{1,2}$ and $u_{2,2}$ as

$$\mu = v_{1,2} + u_{2,2}. \quad (18)$$

μ can be generated from a codebook $\{v_{1,2} + u_{2,2}\}$. Since $v_{1,2}$ and $u_{2,2}$ are encoded with same rate, we assume they are generated from the same codebook, denoted as Φ . Moreover, we design Φ as a set close to the arithmetic plus³. In this way, $\mu = v_{1,2} + u_{2,2}$ can be generated from Φ as well, with the encoding rate, $r_\mu = \beta - \alpha$, identical to that for $v_{1,2}$ and $u_{2,2}$.

Furthermore, as motivated by [4] and [6], we split μ into two parts, μ_1 and μ_2 as

$$\mu \triangleq \{\mu_1, \mu_2\}, \quad (19)$$

each is encoded from a codebook Φ_1 and Φ_2 respectively, which are independent to each other. The encoding rates of these two codebooks are subject to

$$R_{\mu_1} + R_{\mu_2} \approx (\beta - \alpha) \log P, \quad (20)$$

$$r_{\mu_1} + r_{\mu_2} = \beta - \alpha. \quad (21)$$

Hence, Φ can be considered as a product set of Φ_1 and Φ_2 . μ_1 and μ_2 are decoded separately in two parallel channels, μ can therefore be perfectly reconstructed by combining them.

As presented in Table II, μ_1 and μ_2 are respectively superposed on the private symbols transmitted in subband 1 and 2. However, their power $P^{r_{\mu_1} + \beta}$ and $P^{r_{\mu_2} + \beta}$, should not exceed the power constraint P . This constraint can be expressed as

$$r_{\mu_1} \leq 1 - \beta, \quad r_{\mu_2} \leq 1 - \beta. \quad (22)$$

As a consequence, the transmission is subject to the relationship between (21) and (22).

First, in the case of $2(1 - \beta) > \beta - \alpha$, namely $\beta < \frac{2 + \alpha}{3}$, the power of μ_1 and μ_2 does not exceed the per-subband power constraint by simply setting $r_{\mu_1} = r_{\mu_2} = \frac{\beta - \alpha}{2}$. Moreover, we can superimpose a common message $x_{c,1}$ on μ_1 in subband 1 and $x_{c,2}$ on μ_2 in subband 2 using power stated in Table II, which is scaled with P . Second, when $2(1 - \beta) = \beta - \alpha$, the power constraint is still satisfied but no common messages is transmitted since $r_{\mu_1} + \beta = 1$. Third, for the case of $2(1 - \beta) < \beta - \alpha$, namely $\beta > \frac{2 + \alpha}{3}$, the value of r_{μ_1} and r_{μ_2} are bounded by $1 - \beta$ as in (22). Therefore, μ has to be divided into three pieces as

$$\hat{\mu} \triangleq \{\mu_1, \mu_2, \mu_3\}, \quad (23)$$

³ Φ close to arithmetic plus is defined as, if $a \in \Phi$ and $b \in \Phi$, then $a + b \in \Phi$.

with the rates given by

$$R_{\mu_1} = R_{\mu_2} \approx \log P^{r_{\mu_1}}, \quad R_{\mu_3} \approx \log P^{r_{\mu_3}}, \quad (24)$$

$$r_{\mu_1} = r_{\mu_2} = 1 - \beta, \quad r_{\mu_3} = 3\beta - \alpha - 2. \quad (25)$$

The transmission of μ_3 requires an extra channel use in another subband. Next, we will discuss the achievabilities of each point in Figure 2 depending on the requirement of extra channel use.

C. Case I: $\beta \leq \frac{2+\alpha}{3}$ -Achieving Points C and D

In this case, μ is split into two parts and no extra channel use is required. Messages $x_{c,1}$ and $x_{c,2}$ are transmitted provided that $\beta < \frac{2+\alpha}{3}$. The decoding procedure is described as follows.

1) **Stage I-Decode $x_{c,1}$ and $x_{c,2}$:** Revisiting (12) and (13), the received power of $x_{c,1}$ is P at each receiver. Successive interference cancelation (SIC) is selected as the decoding strategy. $x_{c,1}$ is decoded at the first stage treating all the other symbols as noise. Consequently, the rates of $x_{c,1}$ achieved at user 1 and user 2 are $R_{x_{c,1}}^{(1)} = I(x_{c,1}; y_1 | \mathbf{h}_1)$ and $R_{x_{c,1}}^{(2)} = I(x_{c,1}; z_1 | \mathbf{g}_1)$, respectively. These two rates are equal to $\log \frac{P - P^{r_{\mu_1} + \beta}}{P^{r_{\mu_1} + \beta}}$, which is asymptotically $(1 - r_{\mu_1} - \beta) \log P$ for infinite P . Similarly, $x_{c,2}$ achieves the rate $(1 - r_{\mu_2} - \beta) \log P$ in y_2 and z_2 .

2) **Stage II-Decode μ_1 , μ_2 and obtain $\hat{\mu}$:** As μ_1 and μ_2 are independently encoded and sent in subband 1 and 2 respectively, they can be decoded separately at both receivers. After that, μ is obtained by combining them.

In y_1 and z_1 , μ_1 is decoded at the second stage of SIC, where $x_{c,1}$ has been completely subtracted. Treating all the component to the r.h.s. of μ_1 in (12) and (13) as noise, μ_1 is decoded with the rate of $R_{\mu_1}^{(1)} = I(\mu_1; y_1 | \mathbf{h}_1, x_{c,1})$ and $R_{\mu_1}^{(2)} = I(\mu_1; z_1 | \mathbf{g}_1, x_{c,1})$ by user 1 and user 2 respectively. Both quantities are equal to $\log \frac{P^{r_{\mu_1} + \beta} - P^\beta}{P^\beta}$, which is $r_{\mu_1} \log P$ at high SNR. Similarly, μ_2 is decoded with rate $r_{\mu_2} \log P$ in subband 2. After that, μ_1 and μ_2 have been successfully decoded so that μ is completely recovered.

3) **Stage III-Decode u_1 and v_2 :** Employing SIC as the decoding strategy, u_1 and v_2 are decoded from y_1 and z_2 respectively.

Let us introduce a notation, y'_1 , representing the signal after subtracting $x_{c,1}$ and μ_1 as

$$y'_1 = y_1 - h_{1,1}^* (x_{c,1} + \mu_1) \quad (26)$$

$$= \eta_{1,1} + \mathbf{h}_1^H \hat{\mathbf{g}}_1^\perp u_1 + \epsilon_{1,y} \quad (27)$$

$$= \mathbf{h}_1^H \hat{\mathbf{h}}_1 v_{1,2} + \mathbf{h}_1^H \hat{\mathbf{g}}_1^\perp u_1 + \epsilon'_{1,y}, \quad (28)$$

where $\eta_{1,1}$ is given in (16) and $\epsilon'_{1,y}$ results from merging $\epsilon_{1,y}$ and $\mathbf{h}_1^H \hat{\mathbf{h}}_1^\perp v_{1,1}$. Treating $\mathbf{h}_1^H \hat{\mathbf{g}}_1^\perp u_1$ as noise, $v_{1,2}$ can be decoded with the rate $R_{v_{1,2}}^{(1)} = I(v_{1,2}; y'_1 | \mathbf{h}_1, \hat{\mathbf{h}}_1) = \log \frac{P^\beta - P^\alpha}{P^\alpha}$, which is asymptotically equal to $(\beta - \alpha) \log P$ at high SNR. After that, u_1 is seen by subtracting $\mathbf{h}_1^H \hat{\mathbf{h}}_1 v_{1,2}$ from y'_1 as

$$y''_1 = y'_1 - \mathbf{h}_1^H \hat{\mathbf{h}}_1 v_{1,2} = \mathbf{h}_1^H \hat{\mathbf{g}}_1^\perp u_1 + \epsilon'_{1,y}. \quad (29)$$

The rate of u_1 is $R_{u_1}^{(1)} = I(u_1; y''_1 | \mathbf{h}_1, \hat{\mathbf{g}}_1) = \alpha \log P$. Similarly, employing SIC to z_2 results in $R_{v_2}^{(2)} = \alpha \log P$ and $R_{u_2,2}^{(2)} = (\beta - \alpha) \log P$.

4) **Stage IV-Decode v_1 and u_2 :** As μ has been recovered perfectly in Stage II, each user can get access to $v_{1,2} + u_{2,2}$. From (18), the rate of $u_{2,2}$ is obtained as $R_{u_{2,2}}^{(1)} = I(u_{2,2}; \mu | v_{1,2})$, where having the knowledge of $v_{1,2}$ is the prerequisite. As $v_{1,2}$ was successfully decoded at user 1 in Stage III, it can be completely removed from (18), resulting in $R_{u_{2,2}}^{(1)} = (\beta - \alpha) \log P$. Similarly the rate of $v_{1,2}$ at receiver 2 is $R_{v_{1,2}}^{(2)} = I(v_{1,2}; \mu | u_{2,2}) = (\beta - \alpha) \log P$.

After decoding $u_{2,2}$, $u_{2,1}$ is decodable from y_2 . Denoting $y'_2 = y_2 - h_{2,1}^* (x_{c,2} + \mu_2)$, merging $\mathbf{h}_2^H \hat{\mathbf{h}}_2^\perp v_2$ and the noise $\epsilon_{2,y}$ into $\epsilon'_{2,y}$, we have

$$y'_2 = \mathbf{h}_2^H \hat{\mathbf{g}}_2^\perp u_{2,1} + \mathbf{h}_2^H \hat{\mathbf{g}}_2 u_{2,2} + \epsilon'_{2,y}. \quad (30)$$

$u_{2,1}$ is obtained by removing $\mathbf{h}_2^H \hat{\mathbf{g}}_2 u_{2,2}$ from y'_2 , resulting in the rate $R_{u_{2,1}}^{(1)} = I(u_{2,1}; y'_2 | u_{2,2}, \mathbf{h}_2, \hat{\mathbf{g}}_2) = \beta \log P$. Similarly, $v_{1,1}$ is decoded with the rate $R_{v_{1,1}}^{(2)} = \beta \log P$.

To sum up, the DoF achieved in these two subbands are

$$d_1 = \lim_{P \rightarrow \infty} \frac{R_{x_{c,1}}^{(1)} + R_{x_{c,2}}^{(1)} + R_{u_1}^{(1)} + R_{u_{2,1}}^{(1)} + R_{u_{2,2}}^{(1)}}{2 \log P} = \frac{2 + \alpha - \beta}{2}, \quad (31)$$

$$d_2 = \lim_{P \rightarrow \infty} \frac{R_{v_{1,1}}^{(2)} + R_{v_{1,2}}^{(2)} + R_{v_2}^{(2)}}{2 \log P} = \beta, \quad (32)$$

where we assume $x_{c,1}$ and $x_{c,2}$ are intended to user 1 so that point D is achieved. Similarly, point C is achieved if $x_{c,1}$ and $x_{c,2}$ are intended to user 2.

D. Case II: $\beta \geq \frac{2-2\alpha}{3}$ -Achieving Point E

In this case, we remind the reader of the discussion in Section IV-B that μ is split into three parts and an extra channel use is required to transmit μ_3 . Besides, no common message is transmitted.

To achieve point E, we repeat the transmission blocks in (10) and (11) for L times and employ one additional subband, namely subband $2L+1$, to finalize the transmissions of $\mu_{3,i}, i=1,2,\dots,L$, where $\mu_{3,i}$ refers to the third piece of overheard interference generated in subband $2i-1$ and $2i$. The rate of $\mu_{3,i}$ is denoted as $r_{\mu_{3,i}} \log P$ and we assume $r_{\mu_{3,i}} = r_{\mu_3}, i=1,2,\dots,L$. The quality of CSIT in subband $2L+1$ is identical to subband 1.

The transmission in subband $2L+1$ is expressed as

$$s_{2L+1} = [\mu_{3,1}, 0]^T + [\mu_{3,2}, 0]^T + \dots + [\mu_{3,L}, 0]^T + \dots$$

$$\hat{\mathbf{g}}_{2L+1}^\perp u_{2L+1} + \hat{\mathbf{h}}_{2L+1}^\perp v_{2L+1}, \quad (33)$$

with the power and rate allocation presented in Table III. Considering s_{2L+1} and the transmit power, the received signal at user 1 is given as

$$y_{2L+1} = h_{2L+1,1}^* \left(\underbrace{\mu_{3,1}}_P + \underbrace{\mu_{3,2}}_{P^{1-r_{\mu_3}}} + \underbrace{\mu_{3,3}}_{P^{1-2r_{\mu_3}}} + \dots + \underbrace{\mu_{3,L}}_{P^{1-(L-1)r_{\mu_3}}} \right) + \dots$$

$$\underbrace{\mathbf{h}_{2L+1}^H \hat{\mathbf{g}}_{2L+1}^\perp u_{2L+1}}_{P^\alpha} + \underbrace{\mathbf{h}_{2L+1}^H \hat{\mathbf{h}}_{2L+1}^\perp v_{2L+1}}_{P^0} + \epsilon_{2L+1,y}, \quad (34)$$

where all the symbols are decodable using SIC. Specifically, after $\mu_{3,1:i-1}$ are decoded, $\mu_{3,i}$ are decoded treating all the

Symbols	Power	Encoding rate
$\mu_{3,1}$	$P - P^{1-r_{\mu_3}}$	r_{μ_3}
$\mu_{3,2}$	$P^{1-r_{\mu_3}} - P^{1-2r_{\mu_3}}$	r_{μ_3}
$\mu_{3,3}$	$P^{1-2r_{\mu_3}} - P^{1-3r_{\mu_3}}$	r_{μ_3}
\vdots	\vdots	\vdots
$\mu_{3,L}$	$P^{1-(L-1)r_{\mu_3}} - P^\alpha$	r_{μ_3}
u_{2L+1}	$P^\alpha/2$	α
v_{2L+1}	$P^\alpha/2$	α

TABLE III: Power and rate allocation in subband $2L+1$ in case II.

components to the r.h.s. of it in (34) as noise. The rate achieved for $\mu_{3,i}, i < L$ is derived as

$$R_{\mu_{3,i}}^{(1)} = I(\mu_{3,i}; y_{2L+1} | \mathbf{h}_{2L+1}, \mu_{3,1:i-1}) \quad (35)$$

$$= \log \frac{P_{\mu_{3,i}}}{P_{u_{2L+1}} + \sum_{j=i+1}^L P_{\mu_{3,j}}} \quad (36)$$

$$= \log \frac{P^{1-(i-1)r_{\mu_3}} - P^{1-ir_{\mu_3}}}{P^{1-ir_{\mu_3}}} \approx r_{\mu_3} \log P. \quad (37)$$

After decoding $\mu_{3,1:L-1}$, $\mu_{3,L}$ can be decoded treating u_{2L+1} as noise. The rate of $\mu_{3,L}$ is $R_{\mu_{3,L}}^{(1)} \approx \log \frac{P^{1-(L-1)r_{\mu_3}}}{P^\alpha}$, whose pre-log factor is $1 - (L-1)r_{\mu_3} - \alpha$. To make $\mu_{3,L}$ decodable with rate r_{μ_3} , L should satisfy the condition $1 - (L-1)r_{\mu_3} - \alpha = r_{\mu_3}$, resulting in

$$L = \frac{1-\alpha}{r_{\mu_3}} = \frac{1-\alpha}{3\beta-\alpha-2}. \quad (38)$$

Similarly, user 2 can decode $\mu_{3,1:L}$ using SIC.

Consequently, $\hat{\mu}_{1:L}$ can be recovered by collecting and combining $\mu_{1,1:L}$, $\mu_{2,1:L}$ and $\mu_{3,1:L}$. Moreover, all the symbols transmitted from the 1st to $2L$ th subband are decodable using the decoding flow described in Section IV-C. The rates achieved in subband $2i-1$ and $2i$ are $R_{u_{2i-1}}^{(1)} = R_{v_{2i-1}}^{(2)} = \alpha \log P$, $R_{u_{2i,1}}^{(1)} = R_{v_{2i-1,1}}^{(2)} = \beta \log P$ and $R_{u_{2i,2}}^{(1)} = R_{v_{2i-1,2}}^{(2)} = (\beta - \alpha) \log P$.

Besides, u_{2L+1} is decoded in (34) with rate $R_{u_{2L+1}}^{(1)} = \alpha \log P$ at the last stage of SIC after removing all the $\mu_{3,i}$. Similarly, $R_{v_{2L+1}}^{(2)} = \alpha \log P$ is achieved at receiver 2. Finally, we can conclude the *DoF* achieved by each user as

$$d_1 = \lim_{P \rightarrow \infty} \frac{L \times (R_{u_{2i-1}}^{(1)} + R_{u_{2i,1}}^{(1)} + R_{u_{2i,2}}^{(1)}) + R_{u_{2L+1}}^{(1)}}{(2L+1) \log P} = \frac{2+\alpha}{3}, \quad (39)$$

$$d_2 = \lim_{P \rightarrow \infty} \frac{L \times (R_{v_{2i-1}}^{(2)} + R_{v_{2i-1,1}}^{(2)} + R_{v_{2i-1,2}}^{(2)}) + R_{v_{2L+1}}^{(2)}}{(2L+1) \log P} = \frac{2+\alpha}{3}. \quad (40)$$

In the proposed scheme, the transmissions of private symbols in subband 1 and 2 occupy 2β channel uses while transmitting $\hat{\mu}$ requires $\beta - \alpha$ channel uses. Over those channel uses, the sum rate of the private symbols are $4\beta \log P$, resulting in the sum *DoF* $\frac{4\beta}{3\beta-\alpha}$. Revisiting Table I, our new scheme achieves the same sum rate as MAT but using less channel uses. At the same time, it outperforms ZFBF by $2(\beta - \alpha) \log P$ in sum rate with only $\beta - \alpha$ more channel use.

E. "SC+ZF"-Achieving Points A and B

Points A and B can be simply achieved via ZFBF using α channel use and transmitting common message $x_{c,i}$ using the

remaining $1 - \alpha$ channel use in each subband. The transmitted signal in subband 1 is expressed as

$$\mathbf{s}_1 = [x_{c,1}, 0]^T + \hat{\mathbf{g}}_1^\perp u_1 + \hat{\mathbf{h}}_1^\perp v_1, \quad (41)$$

where u_1, v_1 are allocated with power $P^\alpha/2$ and the power of $x_{c,1}$ is $P - P^\alpha$. User 1 observes $x_{c,1}$ plus $\mathbf{h}_1^H \hat{\mathbf{g}}_1^\perp u_1$ because $\mathbf{h}_1^H \hat{\mathbf{h}}_1^\perp v_1$ is drowned by the noise. Treating $\mathbf{h}_1^H \hat{\mathbf{g}}_1^\perp u_1$ as noise, $x_{c,1}$ is decoded at the first stage of SIC with rate $(1 - \alpha) \log P$. After that, u_1 is decoded with the rate α . Similarly, user 2 can decode $x_{c,1}$ and v_1 . The transmission and decoding procedure are applicable to subband 2. As a result, assuming $x_{c,1}$ and $x_{c,2}$ are intended to user 1, the *DoF* are expressed as

$$d_1 = \lim_{P \rightarrow \infty} \frac{R_{x_{c,1}}^{(1)} + R_{x_{c,2}}^{(1)} + R_{u_1}^{(1)} + R_{u_2}^{(1)}}{2 \log P} = 1, \quad (42)$$

$$d_2 = \lim_{P \rightarrow \infty} \frac{R_{v_1}^{(2)} + R_{v_2}^{(2)}}{2 \log P} = \alpha, \quad (43)$$

so that point B is achieved. Point A is achieved if the common messages are assumed intended to user 2.

V. CONCLUSION

This work for the first time in the literature investigates the impact of imperfect CSIT on the *DoF* region of frequency correlated MISO BC. A general two-subband based imperfect CSIT pattern (see Figure 1) is studied. MAT and ZFBF achieve the optimal sum *DoF* for $\alpha=0$ and $\alpha=\beta$ respectively while both of them have *DoF* loss for $0 < \alpha < \beta$. A novel transmission strategy is proposed to improve the performance. It processes as an integration of MAT and ZFBF, where precoding and overheard interference cancellation are combined. The *DoF* region achieved is a function of β and α , enhancing the result for $0 < \alpha < \beta$ and bridging the region of MAT (for $\alpha=0$) and ZFBF (for $\alpha=\beta$).

More details on the achievability and the converse will be provided in our upcoming journal version paper. Besides, we will investigate a more general scenario where the qualities of the four CSIT in Figure 1 are all different. The proposed scheme and the *DoF* region will be extended.

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